Statics Formulas
Gürkan Hoca | Unicourse

Important Unit Conversions

- **Length (US to SI):** 1 ft = 0.3048 m, 1 in = 0.0254 m
- **Length (SI to US):** 1 m $\approx$ 3.281 ft $\approx$ 39.37 in
- **Force (US to SI):** 1 lb (lbf) $\approx$ 4.448 N
- **Force (SI to US):** 1 N $\approx$ 0.2248 lb (lbf)
- **Mass (US to SI):** 1 slug $\approx$ 14.59 kg
- **Mass (SI to US):** 1 kg $\approx$ 0.0685 slug
- **Stress/Pressure (US to SI):** 1 psi $\approx$ 6895 Pa, 1 ksi $\approx$ 6.895 MPa
- **Stress/Pressure (SI to US):** 1 MPa $\approx$ 145 psi
- **Moment/Torque (US to SI):** 1 lb $\cdot$ ft $\approx$ 1.356 N $\cdot$ m
- **Moment/Torque (SI to US):** 1 N $\cdot$ m $\approx$ 0.7376 lb $\cdot$ ft
- **Power (US to SI):** 1 hp $\approx$ 745.7 W
- **Power (SI to US):** 1 kW $\approx$ 1.341 hp

Force Vectors

- **Vector Representation (Cartesian):** $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$
- **Vector Magnitude:** $F = |\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$
- **Unit Vector for vector AB:** $\hat{u}_{AB} = \frac{\vec{AB}}{|AB|} = \frac{\vec{AB}_x}{|AB|}\hat{i} + \frac{\vec{AB}_y}{|AB|}\hat{j} + \frac{\vec{AB}_z}{|AB|}\hat{k}$
- **Direction Cosines:** $\vec{u}_F = \cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k}; \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
- **Dot Product (Scalar Product):** $\vec{A}\cdot\vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$
- **Cross Product (Vector Product):** $\vec{C} = \vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} - (A_xB_z - A_zB_x)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$

Equilibrium of a Particle

- **Condition for Equilibrium:** $\sum\vec{F} = 0$
- **Equilibrium in 2D:** $\sum F_x = 0$ and $\sum F_y = 0$
- **Equilibrium in 3D:** $\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0$
- **Free Body Diagram (FBD):** Diagram showing particle isolated, with all external forces acting ON it (loads, weight, reactions).

Force System Resultants

- **Moment of a Force (Scalar, 2D):** $M_O = Fd$
- **Moment of a Force (Vector, 3D):** $\vec{M}_O = \vec{r}\times\vec{F}$
- **Principle of Moments (Varignon):** Moment of $\vec{F}$ equals sum of moments of its components.
- **Resultant Force:** $\vec{R} = \sum\vec{F}$
- **Resultant Couple Moment:** $\vec{M}_{RO} = \sum\vec{M}_O = \sum(\vec{r}\times\vec{F}) + \sum\vec{M}_C$
- **Reduction of a System:** Equivalent to $\vec{R}$ at point O and $\vec{M}_{RO}$ about O.

Equilibrium of a Rigid Body

- **Conditions for Equilibrium:** $\sum \vec{F} = 0 \quad \text{and} \quad \sum \vec{M}_O = 0$
- **Equilibrium in 2D:** $\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_O = 0$

- **Support Reactions (Common 2D):**
  - **Roller/Rocker:** 1 unknown (force magnitude perpendicular to surface, N).
  - **Pin/Hinge (Smooth):** 2 unknowns (force components, F_x, F_y).
  - **Fixed Support:** 3 unknowns (force components F_x, F_y, and couple moment M).
  - **Cable/Link (Weightless):** 1 unknown (tension magnitude T along cable).
  - **Smooth Contact Surface:** 1 unknown (normal force magnitude N perpendicular to surface).

- **Equilibrium in 3D:** $\sum F_x = 0, \sum F_y = 0, \sum F_z = 0; \quad \sum M_x = 0, \sum M_y = 0, \sum M_z = 0$

- **Support Reactions (Common 3D):**
  - **Ball and Socket:** 3 unknowns (force components F_x, F_y, F_z).
  - **Single Hinge:** 5 unknowns (3 forces F_x, F_y, F_z, 2 moments perpendicular to hinge axis).
  - **Single Journal Bearing:** 4 unknowns (2 forces and 2 moments perpendicular to shaft axis).
  - **Fixed Support:** 6 unknowns (3 forces F_x, F_y, F_z, and 3 couple moments M_x, M_y, M_z).
  - **Cable:** 1 unknown (tension magnitude T).
  - **Smooth Surface Support:** 1 unknown (normal force magnitude N perpendicular to surface).

Structural Analysis

- **Trusses: Basic Assumptions:** Members are all $\Rightarrow$ Two-Force Members (Tension/Compression).
- **Trusses: Method of Joints:** Apply $\sum F_x = 0, \sum F_y = 0$ at joints with ≤ 2 unknowns.
- **Trusses: Method of Sections:** Cut truss ($\leq$ 3 members), apply $\sum F_x = 0, \sum F_y = 0, \sum M_O = 0$ to segment.
- **Frames and Machines:** Contain multi-force members. Disassemble, draw FBD for each member, apply equilibrium: $\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_* = 0$

Internal Forces

- **Internal Loadings (Beam):** Normal Force (N), Shear Force (V), Bending Moment (M) at a section.
- **Sign Convention (on right face):** N: right $\rightarrow$; V: down $\downarrow$; M:CCW $\circlearrowleft$.
- **Relations: Load, Shear, Moment:** $\frac{dV}{dx} = -w(x); \quad \frac{dM}{dx} = V(x)$

Friction

- **Dry Friction (Static):** $f_s \leq f_{s,max} = \mu_s N$
- **Dry Friction (Kinetic):** $f_k = \mu_k N$
- **Impending Motion:** Condition where $f_s = \mu_s N$.
- **Wedges:** Analyzed with FBDs and equilibrium, friction opposes motion/tendency.

Center of Gravity and Centroid

- **Center of Gravity (CG):** $\bar{x} = \frac{\int \tilde{x} dW}{W}, \bar{y} = \frac{\int \tilde{y} dW}{W}, \bar{z} = \frac{\int \tilde{z} dW}{W}$
- **Center of Mass (CM):** $\bar{x} = \frac{\int \tilde{x} dm}{m}, \bar{y} = \frac{\int \tilde{y} dm}{m}, \bar{z} = \frac{\int \tilde{z} dm}{m}$
- **Centroid of Volume:** $\bar{x} = \frac{\int_V \tilde{x} dV}{V}, \bar{y} = \frac{\int_V \tilde{y} dV}{V}, \bar{z} = \frac{\int_V \tilde{z} dV}{V}$
- **Centroid of Area:** $\bar{x} = \frac{\int_A \tilde{x} dA}{A}, \bar{y} = \frac{\int_A \tilde{y} dA}{A}$
- **Centroid of Line:** $\bar{x} = \frac{\int_L \tilde{x} dL}{L}, \bar{y} = \frac{\int_L \tilde{y} dL}{L}$
- **Composite Bodies/Areas/Lines:** $\bar{X} = \frac{\sum \bar{x}_i Q_i}{\sum Q_i}, \bar{Y} = \frac{\sum \bar{y}_i Q_i}{\sum Q_i}, \bar{Z} = \frac{\sum \bar{z}_i Q_i}{\sum Q_i}$ (Q_i=W, V, A, or L)
- **Pappus-Guldinus Theorem I (Surface Area):** $A_{surf} = \theta \bar{y} L$
- **Pappus-Guldinus Theorem II (Volume):** $V = \theta \bar{y} A$

Moment of Inertia (Area)

- **Moment of Inertia about x-axis:** $I_x = \int_A y^2 dA$
- **Moment of Inertia about y-axis:** $I_y = \int_A x^2 dA$
- **Polar Moment of Inertia:** $J_O = \int_A r^2 dA = I_x + I_y$
- **Product of Inertia:** $I_{xy} = \int_A xy dA$
- **Parallel-Axis Theorem (Area):** $I_x = \bar{I}_{x'} + Ad_y^2; \quad I_y = \bar{I}_{y'} + Ad_x^2; \quad I_{xy} = \bar{I}_{x'y'} + Ad_x d_y$
- **Moment of Inertia (Rectangle):** $\bar{I}_{x'} = \frac{1}{12} bh^3$
- **Moment of Inertia (Circle):** $\bar{I}_{x'} = \frac{\pi}{4} r^4$
- **Moment of Inertia (Triangle):** $\bar{I}_{x'} = \frac{1}{36} bh^3$ (about centroidal axis parallel to base b)
- **Radius of Gyration (Area):** $k_x = \sqrt{I_x / A}, \quad k_y = \sqrt{I_y / A}$
- **Moment of Inertia for Composite Areas:** $I_{total} = \sum (\bar{I}_i + A_i d_i^2)$

Virtual Work

- **Virtual Displacement:** Imaginary, infinitesimal displacement ($\delta \vec{r}, \delta \theta$) consistent with constraints.
- **Virtual Work (Force):** $\delta U = \vec{F} \cdot \delta \vec{r}$
- **Virtual Work (Couple Moment):** $\delta U = M \delta \theta$
- **Principle of Virtual Work:** For equilibrium, total virtual work by active external forces/moments is zero: $\delta U = 0$.