Mechanics of Materials Formulas
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Important Unit Conversions

- **Length (US to SI):** 1 ft $= 0.3048$ m, 1 in $= 0.0254$ m
- **Length (SI to US):** 1 m ≈ 3.281 ft ≈ 39.37 in
- **Force (US to SI):** 1 lb (lbf) ≈ 4.448 N
- **Force (SI to US):** 1 N ≈ 0.2248 lb (lbf)
- **Stress/Pressure (US to SI):** 1 psi ≈ 6895 Pa, 1 ksi $= 1000$ psi ≈ 6.895 MPa
- **Stress/Pressure (SI to US):** 1 Pa $\approx 1.450 \times 10^{-4}$psi, 1 MPa ≈ 145 psi ≈ 0.145 ksi
- **Moment/Torque (US to SI):** 1 lb $\cdot$ ft ≈ 1.356 N $\cdot$ m
- **Moment/Torque (SI to US):** 1 N $\cdot$ m ≈ 0.7376 lb $\cdot$ ft
- **Power (US to SI):** 1 hp ≈ 745.7 W (mechanical/electrical)
- **Power (SI to US):** 1 kW $= 1000$ W ≈ 1.341 hp

Concept of Stress

- **Average Normal Stress (Axial Load):** $\sigma = \frac{P}{A}$
- **Average Shear Stress (Direct Shear):** $\tau_{avg} = \frac{V}{A}$
- **Bearing Stress:** $\sigma_b = \frac{P}{A_b}$
- **Allowable Stress Design:** $\sigma_{allow} = \frac{\sigma_{fail}}{FS}$, $\tau_{allow} = \frac{\tau_{fail}}{FS}$

Strain

- **Normal Strain:** $\epsilon = \frac{\delta}{L_0} = \frac{L-L_0}{L_0}$
- **Shear Strain:** $\gamma = \frac{\pi}{2} - \theta'$
- **Small Strain Approximation:** For small γ, $\gamma \approx \tan(\gamma) =$ displacement / original distance.

Mechanical Properties of Materials

- **Stress-Strain Diagram:** Graphical representation of material behavior under load.
- **Hooke's Law (Elastic Region):** $\sigma = E\epsilon$
- **Hooke's Law (Shear):** $\tau = G\gamma$
- **Poisson's Ratio:** $\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$
- **Relation between E, G, :** $G = \frac{E}{2(1+\nu)}$

Axial Load

- **Elastic Deformation (Constant P, A, E):** $\delta = \frac{PL}{AE}$
- **Elastic Deformation (Variable):** $\delta = \int_0^L \frac{P(x)\,dx}{A(x)E}$
- **Principle of Superposition:** Total displacement/stress is the sum of displacements/stresses caused by individual loads (for linear elastic materials).
- **Statically Indeterminate Problems:** Number of unknowns > number of equilibrium equations. Requires compatibility equations (geometry of deformation).
- **Thermal Stress:** $\delta_T = \alpha(\Delta T)L$

Torsion

- **Torsion Formula (Shear Stress):** $\tau = \frac{T\rho}{J}$
- **Maximum Shear Stress (Outer Surface):** $\tau_{max} = \frac{Tc}{J}$
- **Polar Moment of Inertia (Solid Shaft):** $J = \frac{\pi}{2}c^4$
- **Polar Moment of Inertia (Hollow Shaft):** $J = \frac{\pi}{2}(c_o^4 - c_i^4)$
- **Angle of Twist (Constant T, J, G):** $\phi = \frac{TL}{JG}$
- **Angle of Twist (Variable):** $\phi = \int_0^L \frac{T(x)\,dx}{J(x)G}$
- **Power Transmission:** $P = T\omega = 2\pi f T$

Bending

- **Shear and Moment Diagrams:** Graphical representation of internal shear force (V) and bending moment (M) along the beam.
- **Relationships between Load, Shear, Moment:** $\frac{dV}{dx} = -w(x), \quad \frac{dM}{dx} = V(x)$
- **Flexure Formula (Normal Stress):** $\sigma = -\frac{My}{I}$
- **Maximum Bending Stress:** $\sigma_{max} = \frac{Mc}{I} = \frac{M}{S}$
- **Moment of Inertia (Rectangle):** $I_{NA} = \frac{1}{12}bh^3$
- **Moment of Inertia (Circle):** $I_{NA} = \frac{\pi}{4}r^4$
- **Parallel Axis Theorem:** $I = I_{CM} + Ad^2$

Transverse Shear

- **Shear Formula:** $\tau = \frac{VQ}{It}$
- **First Moment of Area (Q):** $Q = \int_{A'} y\,dA = \bar{y}'A'$
- **Shear Stress Distribution (Rectangle):** $\tau_{max} = \frac{3V}{2A}$ (at Neutral Axis)
- **Shear Stress Distribution (I-Beam):** Maximum stress usually occurs at the neutral axis within the web. Flanges carry minimal shear stress.
- **Shear Flow:** $q = \frac{VQ}{I}$

Combined Loadings

- **Principle of Superposition:** Stresses from different load types (axial, torsion, bending, shear) are calculated separately and then added algebraically at a point of interest.
- **Stress Element:** A small cube representing the state of stress ($\sigma_x, \sigma_y, \tau_{xy}$ for plane stress) at a point.
- **Procedure:** 1. Determine internal resultants (P, V, M, T) at the section. 2. Calculate stress components due to each resultant at the point. 3. Combine the stress components on a stress element.

Stress Transformation

- **Plane Stress Transformation Eq ($\sigma_{x'}$):** $\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$
- **Plane Stress Transformation Eq ($\tau_{x'y'}$):** $\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$
- **Principal Stresses:** $\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$
- **Orientation of Principal Planes:** $\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$
- **Maximum In-Plane Shear Stress:** $\tau_{max\,in-plane} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = R$
- **Mohr's Circle:** A graphical method to represent and solve plane stress transformation problems.

Deflection of Beams

- **Elastic Curve:** The deformed shape of the beam's neutral axis under load.
- **Moment-Curvature Relationship:** $\frac{1}{\rho} = \frac{M}{EI}$
- **Governing Differential Equation:** $EI\frac{d^2y}{dx^2} = M(x)$
- **Relationships with Slope, Shear, Load:** $\theta(x) = \frac{dy}{dx}$ (Slope), $EI\frac{d^3y}{dx^3} = V(x)$, $EI\frac{d^4y}{dx^4} = -w(x)$
- **Integration Method:** Integrate $EIy'' = M(x)$ twice. Use boundary conditions (supports) and continuity conditions (internal hinges, connections) to find integration constants.
- **Superposition Method:** Deflection/slope from complex loading is the sum of deflections/slopes from simpler standard loadings (using tables). Valid for linear elastic behavior.
- **Moment-Area Theorem 1:** $\theta_{B/A} = \int_A^B \frac{M}{EI} dx$
- **Moment-Area Theorem 2:** $t_{B/A} = \int_A^B x\frac{M}{EI} dx$

Buckling of Columns

- **Critical Load (Euler's Formula):** $P_{cr} = \frac{\pi^2 EI}{L_e^2}$
- **Effective Length ($L_e = KL$):** $L_e = KL$. K depends on end conditions:
  Pinned-Pinned: $K = 1.0$
  Fixed-Fixed: $K = 0.5$
  Fixed-Pinned: $K = 0.7$
  Fixed-Free: $K = 2.0$
- **Critical Stress (σ_{cr}):** $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L_e/r)^2}$ ($r = \sqrt{I/A}$ radius of gyration)
- **Applicability of Euler's Formula:** Valid only if $\sigma_{cr} \leq \sigma_{pl}$ (proportional limit / yield strength).