Engineering Dynamics Formulas
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Important Unit Conversions

- **Length:** 1 m ≈ 3.281 ft, 1 ft = 0.3048 m
- **Mass:** 1 kg ≈ 0.06852 slug, 1 slug ≈ 14.59 kg
- **Force:** 1 N ≈ 0.2248 lb (lbf), 1 lb (lbf) ≈ 4.448 N
- **Acceleration:** 1 m/s^2 ≈ 3.281 ft/s^2, 1 ft/s^2 = 0.3048 m/s^2
- **Acceleration of Gravity (g):** $g \approx 9.81$ m/s^2 ≈ 32.2 ft/s^2
- **Energy / Work:** 1 J ≈ 0.7376 ft · lb, 1 ft · lb ≈ 1.356 J
- **Power (Mechanical):** 1 W ≈ 0.001341 hp ≈ 0.7376 ft · lb/s
- **Power (Mechanical):** 1 hp ≈ 745.7 W ≈ 550 ft · lb/s
- **Moment / Torque:** 1 N · m ≈ 0.7376 lb · ft, 1 lb · ft ≈ 1.356 N · m
- **Moment of Inertia (Mass):** 1 kg · m^2 ≈ 0.7376 slug · ft^2
- **Pressure / Stress:** 1 Pa = 1 N/m^2 ≈ 1.450 × 10^{-4} psi
- **Pressure / Stress:** 1 psi = 1 lb/in^2 ≈ 6895 Pa

Kinematics of a Particle

- **Rectilinear Kinematics: Continuous Motion:** $v = \frac{ds}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2}, \quad a\,ds = v\,dv$
- **Rectilinear Kinematics: Constant Acceleration:** $v = v_0 + a_c t, \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2, \quad v^2 = v_0^2 + 2a_c(s - s_0)$
- **2D Motion: x-y-z coords.:** $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, \quad \vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}, \quad \vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$
- **2D Motion: n-t coords.:** $\vec{v} = v\hat{u}_t, \quad \vec{a} = a_t\hat{u}_t + a_n\hat{u}_n \quad (a_t = \dot{v},\ a_n = v^2/\rho)$
- **2D Motion: Polar coords. :** $\vec{v} = v_r\hat{u}_r + v_\theta\hat{u}_\theta,\ \vec{a} = a_r\hat{u}_r + a_\theta\hat{u}_\theta\ (v_r = \dot{r}, v_\theta = r\dot{\theta}, a_r = \ddot{r} - r\dot{\theta}^2, a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta})$
- **Absolute Dependent Motion (Two Particles):** questions with ropes. $\sum L = \text{const} \Rightarrow \sum v = 0 \Rightarrow \sum a = 0$
- **Relative Motion:** $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}, \quad \vec{v}_B = \vec{v}_A + \vec{v}_{B/A}, \quad \vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

Kinetics of a Particle: Force and Acceleration

- **Equation of Motion (EoM) (x-y-z):** $\sum \vec{F} = m\vec{a}; \quad \sum F_x = ma_x,\ \sum F_y = ma_y,\ \sum F_z = ma_z$
- **EoM (n-t):** $\sum F_t = ma_t,\ \sum F_n = ma_n,\ \sum F_b = 0 \quad (a_t = \dot{v}, a_n = v^2/\rho)$
- **EoM (Polar):** $\sum F_r = ma_r,\ \sum F_\theta = ma_\theta,\ \sum F_z = ma_z \quad (a_r = \ddot{r} - r\dot{\theta}^2, a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}, a_z = \ddot{z})$

Kinetics of a Particle: Work and Energy

- **Work of a Force:** $U_{1-2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$
- **Principle of Work and Energy:** $T_1 + \sum U_{1-2} = T_2 \quad (T = \frac{1}{2}mv^2)$
- **Potential Energy (Gravity):** $V_g = Wy = mgy$
- **Potential Energy (Elastic Spring):** $V_e = \frac{1}{2}ks^2$
- **Conservation of Energy:** $T_1 + V_1 + (U_{1-2})_{nc} = T_2 + V_2 \quad (V = V_g + V_e)$
- **Power and Efficiency:** $P = \frac{dU}{dt} = \vec{F} \cdot \vec{v}, \quad \epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$

Kinetics of a Particle: Impulse and Momentum

- **Principle of Linear Impulse and Momentum:** $m\vec{v}_1 + \sum \int_{t_1}^{t_2} \vec{F} dt = m\vec{v}_2$
- **Conservation of Linear Momentum (System):** If $\sum \vec{F}_{ext} = 0$, then $\sum(m\vec{v})_1 = \sum(m\vec{v})_2$
- **Impact (coef. of restitution):** $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} \quad (0 \leq e \leq 1)$
- **Angular Impulse and Momentum (Particle):** $(\vec{H}_O)_1 + \sum \int_{t_1}^{t_2} \vec{M}_O dt = (\vec{H}_O)_2 \quad (\vec{H}_O = \vec{r} \times m\vec{v})$

Planar Kinematics of a Rigid Body

- **Types of Planar Motion:** Translation, Rotation about a Fixed Axis, General Plane Motion
- **Rotation About a Fixed Axis:** $v = \omega r, \quad a_t = \alpha r, \quad a_n = \omega^2 r \quad (\omega = \dot{\theta}, \alpha = \ddot{\theta})$
- **Absolute Motion Analysis:** Using geometric constraints to relate position, velocity, acceleration.
- **Relative Velocity (General Plane Motion):** $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = \vec{v}_A + (\vec{\omega} \times \vec{r}_{B/A})$
- **Ins. Center of Zero Vel. (IC Method):** draw perp. axes to velocities. Intersection is IC. Then $v = \omega r_{IC}$
- **Relative Acceleration (General Plane Motion):** $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} = \vec{a}_A + (\vec{\alpha} \times \vec{r}_{B/A}) - \omega^2 \vec{r}_{B/A}$
- **Relative Velocisy with Rotating Axes:** $\vec{v}_B = \vec{v}_A + (\vec{v}_{B/A})_{xyz} + \vec{\Omega} \times \vec{r}_{B/A}$
- **Relative Acceleration with Rotating Axes (Corriolis):**
- : $\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_{xyz} + \dot{\vec{\Omega}} \times \vec{r}_{B/A} + 2\vec{\Omega} \times (\vec{v}_{B/A})_{xyz} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/A})$

Planar Kinetics of a Rigid Body: Force and Acceleration

- **Moment of Inertia (Planar):** $I = \int r^2 dm$
- **Parallel-Axis Theorem:** $I = I_G + md^2$
- **Equations of Motion: Translation:** $\sum F_x = m(a_G)_x, \quad \sum F_y = m(a_G)_y$
- **Equation of Motion: Rotation about G or O:** $\sum M_G = I_G \alpha$ OR $\sum M_O = I_O \alpha$
- **Equations of Motion: General Plane Motion:**
- : $\sum F_x = m(a_G)_x, \ \sum F_y = m(a_G)_y, \ \sum M_G = I_G \alpha$ or $\sum M_O = I_O \alpha$ or $\sum M_P = \sum (M_k)_P$

Planar Kinetics of a Rigid Body: Work and Energy

- **Kinetic Energy (Rigid Body):** $T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$
- **Work of a Force (Rigid Body):** $U_F = \int \vec{F} \cdot d\vec{r}_P$
- **Work of a Couple Moment:** $U_M = \int_{\theta_1}^{\theta_2} M d\theta$
- **Principle of Work and Energy (Rigid Body):** $T_1 + \sum U_{1-2} = T_2$
- **Conservation of Energy (Rigid Body):** $T_1 + V_1 + (U_{1-2})_{nc} = T_2 + V_2$

Planar Kinetics of a Rigid Body: Impulse and Momentum

- **Linear Impulse and Momentum (Rigid Body):** $m(\vec{v}_G)_1 + \sum \int_{t_1}^{t_2} \vec{F} dt = m(\vec{v}_G)_2$
- **Angular Impulse and Momentum (Rigid Body, about G):** $(H_G)_1 + \sum \int_{t_1}^{t_2} M_G dt = (H_G)_2$ where $H_G = I_G \omega$
- **Conservation of Momentum (Linear and Angular):** If no impulse (linear/angular), then corresponding momentum conserved.
- **Eccentric Impact:** Combines principles of impulse/momentum (linear and angular) and coefficient of restitution.

Three-Dimensional Kinematics of a Rigid Body

- **Rotation About a Fixed Point:** Described using rotation matrices or Euler angles. $\vec{v} = \vec{\Omega} \times \vec{r}$
- **General Motion:** $\vec{v}_B = \vec{v}_A + \vec{\Omega} \times \vec{r}_{B/A}, \quad \vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/A})$
- **Relative Motion Analysis (Rotating Axes):** $\vec{v}_B = \vec{v}_A + (\vec{v}_{B/A})_{xyz} + \vec{\Omega} \times \vec{r}_{B/A}$
- **Relative Acceleration Analysis (Rotating Axes):** $\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_{xyz} + \dot{\vec{\Omega}} \times \vec{r}_{B/A} + 2\vec{\Omega} \times (\vec{v}_{B/A})_{xyz} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/A})$

Three-Dimensional Kinetics of a Rigid Body

- **Moments and Products of Inertia:** $I_{xx} = \int (y^2 + z^2) dm, \quad I_{xy} = -\int xy\, dm, \ldots$
- **Angular Momentum (3D, Principal Axes):** $\vec{H}_G = I_{xx}\omega_x \hat{i} + I_{yy}\omega_y \hat{j} + I_{zz}\omega_z \hat{k}$
- **Angular Momentum (3D, General):** $\vec{H}_O = \mathbf{I}_O \vec{\omega} \quad$ ($\mathbf{I}_O$ is inertia tensor)
- **Equations of Motion (3D):** $\sum \vec{F} = m\vec{a}_G, \quad \sum \vec{M}_G = (\dot{\vec{H}}_G)_{Gxyz} + \vec{\Omega} \times \vec{H}_G$
- **Euler's Equations (Principal Axes, Body Frame):** $\sum M_x = I_{xx}\dot{\omega}_x - (I_{yy} - I_{zz})\omega_y \omega_z$, (and cyclic permutations for y, z)

Vibrations

- **Undamped Free Vibration:** $m\ddot{x} + kx = 0, \quad \omega_n = \sqrt{k/m}, \quad x(t) = A\sin(\omega_n t) + B\cos(\omega_n t)$
- **Period and Natural Frequency:** $T = 2\pi/\omega_n, \quad f_n = 1/T = \omega_n/(2\pi)$
- **Energy Method (Vibration):** $T + V =$ constant $\Rightarrow \frac{d}{dt}(T + V) = 0$
- **Damped Free Vibration:** $m\ddot{x} + c\dot{x} + kx = 0$
- **Critical Damping and Damping Ratio:** $c_c = 2m\omega_n = 2\sqrt{km}, \quad \zeta = c/c_c$
- **Damped Natural Frequency (Underdamped, $\zeta < 1$):** $\omega_d = \omega_n\sqrt{1 - \zeta^2}$
- **Forced Vibration (Harmonic Excitation):** $m\ddot{x} + c\dot{x} + kx = F_0\cos(\omega t)$
- **Magnification Factor (Steady State Amp.):** $M.F. = \frac{X}{X_{st}} = \frac{1}{\sqrt{(1-(\omega/\omega_n)^2)^2 + (2\zeta\omega/omega_n)^2}} \quad (X_{st} = F_0/k)$
- **Resonance:** Occurs when driving freq $\omega \approx \omega_n$, leading to large amplitudes (esp. for small ζ).